Comment on "Nonrelativistic electromagnetic surface waves: Dispersion properties in a magnetized dusty electron-positron plasma"

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(Received 2 February 2004; published 15 November 2004)

The theory of electromagnetic surface modes propagating along the planar interface between dusty electron-positron plasma and vacuum is reexamined by the conventional matching method of boundary conditions. It is shown that in a magnetoplasma the direct use of specular reflection method is not appropriate and the derivations for the TM-mode dispersion relation [Phys. Rev. E **61**, 4357 (2000)] are incorrect.

DOI: 10.1103/PhysRevE.70.058401 PACS number(s): 52.35.Hr, 52.35.Lv, 52.27.Lw

I. INTRODUCTION

Recently, Cho *et al.* [1] published a paper dealing with nonrelativistic electromagnetic surface modes in a magnetized dusty electron-positron plasma. Surface wave propagation in a magnetized dusty plasma was previously investigated by many authors (e.g., see Refs. [2–4]). They dealt with the problems by the matching method of boundary conditions. Cho and Lee previously studied an unmagnetized plasma by employing both a matching and specular reflection method [5] and they showed there that both methods lead to the same results. Our objective here is to show that under the reflection $x \to -x$, $v_x \to -v_x$ the governing equaions in Ref. [1] do not remain invariant so that the use of the specular reflection procedure in a magnetized plasma is incorrect. The error of Cho *et al.* is due to the application of the specular reflection method. We derive the correct dispersion relation by the matching method of boundary conditions to distinguish with that in Ref. [1]. We also derive the expressions for the current components correctly to show that the similar results obtained in Ref. [1] are wrong.

II. FLUID MODEL AND BASIC EQUATIONS

The same fluid model is used as in Ref. [1], which consists of the momentum and continuity equations for electrons and positrons as well as Maxwell equations not ignoring the electron inertia and displacement current. The negatively charged dust grains which can effectively collect the electrons and positrons from the background are considered to be point charges and their sizes are assumed to be much smaller than the electron Debye length and the distance between the plasma particles. In the steady state we have

$$n_{0+} = n_{0-} + Z_d n_{d0}, \tag{1}$$

where the subscript "0" stands for equilibrium number density for α species ($\alpha = +,-$, and d for positron, electron, and dust). The parameter $\delta = n_{0-}/n_{0+}$ measures the charge imbalance in the plasma, with the remainder of the charge residing on the dust particles, so that the total system is charge neutral. We assume for simplicity that the charge on the dust grains is not affected by the wave; i.e., we neglect the dust charging effects [6,7]. Now assuming the plasma to occupy the half space $x > 0$ bounded by vacuum $x < 0$ with the planar sharp interface at $x = 0$, the external magnetic field B_0 along the z axis and the time dependence $\sim \exp(-i\omega t)$ we have the basic equations

$$\vec{v}_\alpha = \frac{i\omega}{\omega^2 - \Omega_\alpha^2}\left[\frac{q_\alpha}{m_\alpha}\vec{E} + \frac{i\Omega_\alpha q_\alpha}{m_\alpha \omega}\vec{E} \times \hat{y}\right], \tag{2}$$

$$n_\alpha = \frac{n_{0\alpha}}{i\omega}\nabla \cdot \vec{v}_\alpha, \tag{3}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi e(n_+ - n_-), \tag{4}$$

$$\vec{\nabla} \times \vec{E} = \frac{i\omega}{c}\vec{B}, \tag{5}$$

$$\vec{\nabla} \times \vec{B} = -\frac{i\omega}{c}\vec{E} + \frac{4\pi e}{c}(n_{0+}\vec{v}_+ - n_{0-}\vec{v}_-), \tag{6}$$

where the subscript α stands for electron ($-$) and positron ($+$). Other notations are standard. First we wish to derive the dispersion relation for TM surface modes by the conventional matching method of boundary conditions.

Taking curl of Eq. (6) and using Eq. (5) we obtain

$$\left(c^2\nabla^2 + \omega^2 - \frac{\omega^2(\omega_{p+}^2 + \omega_{p-}^2)}{\omega^2 - \Omega^2}\right)\hat{y} \cdot \vec{\nabla} \times \vec{E} + \frac{i\omega\Omega}{\omega^2 - \Omega^2}(\omega_{p+}^2 - \omega_{p-}^2)\vec{\nabla} \cdot \vec{E} = 0, \tag{7}$$

where $\Omega = eB_0/cm$ is the cyclotron and $\omega_{p(+,-)}$ are the plasma frequencies ($m_+ = m_- = m$). From Eqs. (4) and (5) eliminating $\vec{B}$ using Eq. (2) we obtain

$$(\omega^2 - \Omega^2 - \omega_{p+}^2 - \omega_{p-}^2)\vec{\nabla} \cdot \vec{E} = \frac{i\Omega}{\omega}(\omega_{p+}^2 - \omega_{p-}^2)\hat{y} \cdot \vec{\nabla} \times \vec{E}. \tag{8}$$

Equations (7) and (8) are two coupled equations for our magnetized plasma, which give the wave equation

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$$[(c^2\nabla^2+\omega^2\lambda)\lambda-\Lambda^2]\begin{pmatrix}\hat{y}\cdot\vec{\nabla}\times\vec{E}\\ \vec{\nabla}\cdot\vec{E}\end{pmatrix}=0. \tag{9}$$

Here

$$\lambda=1-\frac{\omega_{p+}^2+\omega_{p-}^2}{\omega^2-\Omega^2},\Lambda=\frac{\Omega(\omega_{p+}^2-\omega_{p-}^2)}{\omega^2-\Omega^2}.$$

Assuming the wave fields to vary as $\exp(ikz-i\omega t)$ we have, from Eq. (9),

$$\hat{y}\cdot\vec{\nabla}\times\vec{E}\equiv ikE_x-\frac{\partial E_z}{\partial x}=\begin{cases}A_1e^{\alpha_1 x} & (x<0),\\ A_2e^{-\alpha_2 x} & (x>0),\end{cases} \tag{10}$$

$$\vec{\nabla}\cdot\vec{E}\equiv ikE_z+\frac{\partial E_x}{\partial x}=\frac{i\Omega(\omega_{p+}^2-\omega_{p-}^2)}{\omega(\omega^2-\Omega^2)\lambda}\begin{cases}A_1e^{\alpha_1 x} & (x<0),\\ A_2e^{-\alpha_2 x} & (x>0),\end{cases} \tag{11}$$

where

$$\alpha_1=\left(k^2-\frac{\omega^2}{c^2}\right)^{1/2},\ \alpha_2=\left(k^2-\frac{\omega^2\lambda}{c^2}-\frac{\Lambda^2}{c^2\lambda}\right)^{1/2}. \tag{12}$$

Equations (10) and (11) are the simultaneous equations for E_x and E_z giving

$$E_z=\begin{cases}\left(\alpha_1+\frac{k\Lambda}{\omega\lambda}\right)\frac{A_1}{k^2-\alpha_1^2}e^{\alpha_1 x} & (x<0),\\ \left(-\alpha_2+\frac{k\Lambda}{\omega\lambda}\right)\frac{A_2}{k^2-\alpha_2^2}e^{-\alpha_2 x} & (x>0).\end{cases} \tag{13}$$

Clearly, we need two boundary conditions to determine two integration constants A_1 and A_2 which are the continuity of the tangential component of $\vec{E}$ as well as the normal component of $\vec{B}$ across the interface—i.e.,

(i) $E_z(x=0^+)=E_z(x=0^-)$,

(ii) $B_y(x=0^+)=B_y(x=0^-)$.

Thus the above boundary conditions lead to the following dispersion relation for surface TM modes in a dusty electron-positron magnetoplasma:

$$(\alpha_1+\alpha_2)[\omega(\alpha_1\alpha_2-k^2)\lambda+k\Lambda(\alpha_2-\alpha_1)]=0. \tag{14}$$

It is straightforward to show that the first factor of Eq. (14) must be nonzero, whereas equating the second factor to zero we have

$$\left[\left(K-\frac{\overline{\omega}\overline{\lambda}}{\delta}+\frac{\overline{\Lambda}^2}{\overline{\lambda}}\right)^{1/2}\left(K-\frac{\overline{\omega}}{\delta}\right)^{1/2}-K\right]\overline{\lambda}\sqrt{\overline{\omega}}+\overline{\Lambda}\sqrt{K}\times\left[\left(K-\frac{\overline{\omega}\overline{\lambda}}{\delta}+\frac{\overline{\Lambda}^2}{\overline{\lambda}}\right)^{1/2}-\left(K-\frac{\overline{\omega}}{\delta}\right)^{1/2}\right]=0. \tag{15}$$

Here we have used the following dimensionless parameters

$$K=c^2k^2/\omega_{p-}^2,\ \ \overline{\omega}=\omega^2/\omega_{p+}^2,\ \ \overline{\Omega}=\Omega^2/\omega_{p+}^2$$

$$\delta=\frac{n_{0-}}{n_{0+}},\ \ \overline{\lambda}=1-\frac{1+\delta}{\overline{\omega}-\overline{\Omega}},\ \ \overline{\Lambda}=\frac{(1-\delta)\sqrt{\overline{\Omega}}}{\overline{\omega}-\overline{\Omega}}.$$

By making the correspondence $\overline{\Omega}\to 0,\omega_{p+}\to 0,\delta\to 1$ one can recover the dispersion relation (29) in Ref. [5] for an unmagnetized cold electron plasma. The dispersion relation (15) as obtained by the conventional matching method of boundary conditions is not identical with that in Ref. [1] obtained by the specular reflection method. In the following section we explain how the specular reflection procedure fails for a magnetized plasma and adopt some correct derivations to show that the results shown to obtain the dispersion relation in Ref. [1] are incorrect.

III. DISCUSSION

Here we first show that the corresponding derivations in Ref. [1] are incorrect. From the linearized momentum and continuity equations **(**Eqs. (9) and (10) in Ref. [1]**)** we obtain

$$v_{\alpha x}=\frac{iq_\alpha\omega}{m_\alpha(\omega^2-\Omega_\alpha^2)}\left(E_x-i\frac{\Omega_\alpha}{\omega}E_z\right), \tag{16a}$$

$$v_{\alpha z}=\frac{iq_\alpha\omega}{m_\alpha(\omega^2-\Omega_\alpha^2)}\left(E_z+i\frac{\Omega_\alpha}{\omega}E_x\right), \tag{16b}$$

$$n_\alpha=\frac{n_{0\alpha}}{\omega}\vec{k}\cdot\vec{v}_\alpha. \tag{17}$$

Substituting the above velocity expressions in the current $\vec{J}=\Sigma q_\alpha n_\alpha\vec{v}_\alpha$ we easily obtain

$$J_x=\frac{i\omega}{4\pi(\omega^2-\Omega^2)}\left[(\omega_{p+}^2+\omega_{p-}^2)E_x-\frac{i\Omega}{\omega}(\omega_{p+}^2-\omega_{p-}^2)E_z\right], \tag{18a}$$

$$J_z=\frac{i\omega}{4\pi(\omega^2-\Omega^2)}\left[(\omega_{p+}^2+\omega_{p-}^2)E_z+\frac{i\Omega}{\omega}(\omega_{p+}^2-\omega_{p-}^2)E_x\right], \tag{18b}$$

for the current components responsible for the electromagnetic (em) wave. Cho *et al.* omitted the terms proportional to Ω which arise due to the static magnetic field B_0. They simply showed $J_x\sim E_x$ and $J_z\sim E_z$ without any reason. Expressions (18a) and (18b) when substituted in the Fourier-transformed Maxwell equations **(**see Eqs. (8a)–(8c) in Ref. [1]**)** lead to the following system in matrix form:

$$\begin{pmatrix}k_x & -k_z & \omega/c\\ \xi & -\lambda & ck_z/\omega\\ \lambda & \xi & ck_x/\omega\end{pmatrix}\begin{pmatrix}E_z\\ E_x\\ B_y\end{pmatrix}=\begin{pmatrix}0\\ 0\\ -\frac{iac}{\pi\omega}\end{pmatrix}, \tag{19}$$

where

$$\xi = -\frac{i\Omega(\omega_{p+}^2 - \omega_{p-}^2)}{(\omega^2 - \Omega^2)\omega}. \tag{20}$$

Note that the term ξ arises due to the presence of the trems proportional to Ω in the expressions for J_x and J_z [Eqs. (18a) and (18b)] and which, in general, cannot be neglected in a magnetized plasma. On the other hand, in absence of these terms the matrix equation (21) simply becomes

$$\begin{pmatrix} k_x & -k_z & \omega/c \\ 0 & -\lambda & ck_z/\omega \\ \lambda & 0 & ck_x/\omega \end{pmatrix}\begin{pmatrix} E_z \\ E_x \\ B_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\dfrac{iac}{\pi\omega} \end{pmatrix}. \tag{21}$$

But in Ref. [1] as mentioned above, the expressions for J_x and J_Z do not contain such terms proportional to Ω, yet Cho *et al.* displyed their 3×3 matrix **(**Eq. (13) in Ref. [1]**)** erroneously with no zero term and incorrect elements in the (2, 1) (2, 2), (3, 1), (3, 2) positions. Furthermore, solving the correct form [Eq. (21)] one can find after a straightforwrd complex analysis **(**picking up the residue at the simple pole $k_x = i\eta$, $\eta = [k_z^2 - \omega^2(\xi^2+\lambda^2)/(c^2\lambda)]^{1/2}$**)** that

$$B_y(x = 0^+) = a + \frac{ia\xi k_z}{\eta\lambda}, \tag{22}$$

where ξ is proportaional to Ω and it vanishes when there is no external magnetic field, which shows that the normal component of the magnetic field is discontinuous across the interface $[B_y(x=0^-)=a]$. Here B_y can be discontinuous only when there is a surface current on the interface. A static magnetic field alone can not give rise any physical mechanism to produce such surface current. This is also clear from the fact that if we use the divergence theorem on the Maxwell equation $\nabla\cdot\vec{B}=0$, we would always have $B_y(x=0^+) = B_y(x=0^-)$. Thus, we arrive at the absurd result. That is, the direct use of the specular reflection procedure in a magnetized plasma is not appropriate. It is now straightforward to show that under the reflection $x\to -x, v_x \to -v_x$ Eq. (16a) does not remain invariant, because $E_z(-x)=E_z(x)$ and $E_x(-x) = -E_x(x)$ **(**see Eq. (7) in Ref. [1]**)**. This violation is due to the term proportional to Ω that omitted in Ref. [1], and that can be removed in an unmagnetized case [5].

IV. CONCLUSIONS

In our above analysis we have reexamined the results obtained in Ref. [1] by employing the conventional matching method of boundary conditions. We have shown that under the reflection the governing equation (2) does not remain invariant and as such the specular reflection procedure fails in such a magnetized plasma. In Ref. [1] the authors used this approach in the magnetized case without checking the invariance of the governing equations under the reflection, which we have shown finally led to erronious results. We have also demonstrated that the derivations for the TM dispersion in Ref. [1] are incorrect. The correct form of the dispersion relation is derived which is in good agreement with that in Ref. [5] for an unmagnetized cold electron plasma case. The different mode of propagation can be analyzed by solving our dispersion relation (15) both analytically and numerically, which will be communicated in the near future.

ACKNOWLEDGMENT

A.P.M. wishes to thank the Council of Scientific and Industrial Research, Government of India, for support.

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